

## Lease Sales Tax Treatment in Illinois

CAVEAT: State tax guidelines matter little (e.g., NY Pub 839, Ill ST-9-LSE). What matters is the state's tax code. However, for sake of simplicity, we'll defer to the tax guidelines as that is the commonly used reference among lay people.

Sales Tax levied on car leases in Ill and NY are computed in the same way. Both states multiply qualifying DAS fees by  $\frac{\tau}{1-\tau}$  when the sales tax on those qualifying fees are capitalized. Otherwise, it makes no sense to the extent that it results in tax levied on tax. We'll examine only those leases that capitalize sales tax and those that use money factors as very few fund providers (e.g., Ally Bank, Ford Motor Credit) use an interest rate to compute lease payments.

**If you are rolling (i.e., capitalizing) NY and Illinois sales tax into your lease and are paying taxable DAS fees in which the sales tax is also capitalized, you are paying sales tax on sales tax.** As far as I know, the constitutionality of this has never been challenged. In fact, I'm betting that few NY and Illinois lessors and lessees realize that sales tax is being levied on sales tax in such instances. Here's proof using an Illinois dealership's lease worksheet data.

### EXAMPLE I

	Source:	IFS	Monthly Payment:	417.8E	
Product:	Lease	Contract Date:	12/10/2018	Security Deposit:	0.0C
Odometer:	4	Vehicle MSRP:	54030.00	Due at Signing:	2500.0C
Original MSRP:	54030.00	Gas Guzzler Tax:	0.00	Rebate:	1500.0C
Base Invoice:	51032.00	Package Discount:	0.00	Cash Offset:	0.0C
Dealer Cash:	0.00	Add Options:	0.00	Trade-In Down:	0.0C
Vehicle Price:	45500.00	Remove Options:	0.00	Cash Due:	1000.0C
Soft Adds:	0.00	Deduct Options:	0.00	Actual Miles:	1000C
Service Contract:	0.00	Dealer Installed:	0.00	Standard Miles:	1500C
Gap Insurance:	0.00	Residual Basis:	54030.00	Initial Charge:	0.1C
Sales Tax (C):	1514.48	Book (%):	54.000%	Excess Charge:	0.2E
Flat Tax:	15.00	Net Residual (%):	57.000%	Total Sales Tax:	1514.4E
Misc Advance Tax:	0.00	Net Residual (\$):	30797.10	Sales Tax Rate:	8.250%
DMV Misc Fees:	396.94	Sell Rate:	0.00035	Monthly Use Tax:	0.00
Credit Insurance:	0.00	Published:	0.00035	Monthly Use Tax Rate:	0.000%
Acquisition Fee (C):	700.00	Credit Tier:	Tier 1	Cap Reduction Tax:	0.00
VSI Fee (C):	0.00	Front-End Profit:	-5532.00	Cap Reduction Tax Rate:	0.000%
Other:	0.00	Back-End Profit:	0.00	1st Payment Date:	12/10/2018
Trade-In Balance:	0.00	Reserve Profit:	150.00	2nd Payment Date:	01/10/2019
Gross Cap Cost:	48126.42	Total Profit:	-5382.00		
Cash Down:	582.15	DMV Fees:	C=Cap		
Cap Reduction:	582.15		U=Upfront		
Rebate (CR):	1500.00	Documentation Fee (C):	175.94		
Trade-In Down (CR):	0.00	Registration Fee (C):	101.00		
Dealer Cash (CR):	0.00	License Fee (U):	0.00		
Adjusted Cap Cost:	46044.27	Title Fee (C):	95.00		
Rebate:	1500.00	Lien Fee (U):	0.00		
ACV:	0.00	Electronic Filing Fee (C):	25.00		
Allowance:	0.00	Total:	396.94		
Payoff:	0.00				
Incentives Applied:					
2019 INFINITI QX60 Luxe AWD Special Financing:	0.00				
2019 INFINITI QX60 Winter Bonus Lease Offer:	1500.00				

Summarizing the dealer's lease worksheet, we have:

$$\tau = \text{Sales Tax Rate} = 8.250\%$$

$$F = \text{Money Factor} = 0.00035$$

$$N = \text{Term (months)} = 39$$

$$S = \text{Sell Price} = 45,500.00$$

$$A = \text{Capped Fees subject to tax (includes flat tax)} = 1,111.94$$

$$C = \text{Capped Fees not subject to tax} = 1,514.48 \text{ (sales tax)}$$

$$D = \text{Cash Cap Reduction plus Rebate Cap Reduction} = 582.15 + 1500.00 = 2,082.15$$

$$R = \text{Residual Value} = 57\% \times 54,030 = 30,797.10$$

$$P_L = \text{Contractual Monthly Lease Payment} = 417.85$$

$$\text{DAS} = 582.15 + 417.85 = 1000$$

$$U = \text{DAS fees, other than taxable cap reductions, subject to sales tax} = 0$$

We'll calculate the following:

$$P_B = \text{Base Payment (Excludes Sales Tax)} = 378.48$$

$$T_B = \text{Total Payment Sales Tax Liability} = 1,327.26$$

$$T_D = \text{Sales Tax on Cash Cap Reduction and Rebate} = 187.22$$

$$T = \text{Total Sales Tax Liability} = 1,514.48$$

$$P_L = \text{Contractual Monthly Lease Payment}$$

NOTE: The only purpose of the base payment,  $P_B$ , is to compute the payment sales tax.

## METHOD I

$$\text{Base payment (excludes sales tax)} : P_B = F(S + A - D + R) + \frac{(S + A - D - R)}{N}$$

Substituting the assigned values, we get  $P_B = 378.48$

Compute the total base payments,  $K$ :

$$\begin{aligned} K &= P_B N \\ &= 378.48 \times 39 \\ &= 14,760.72 \end{aligned}$$

Adding the payment sales tax,  $T_B$ , to the total base payments,  $K$ , and then multiplying the sum by the sales tax rate,  $\tau$ , we have the formula...

$$T_B = (K + T_B) \tau$$

$$T_B = K\tau + T_B\tau \dots \text{the term } T_B\tau \text{ reflects sales tax levied on sales tax}$$

Solving for  $T_B$ , we get

$$T_B = K \frac{\tau}{1 - \tau}, \text{ where } \frac{\tau}{1 - \tau} = \text{Sales Tax Factor} \dots$$

Substituting the assigned values, we have

$$\begin{aligned} T_B &= (14,760.72) \frac{0.08250}{1 - 0.08250} \\ &= \$1,327.26 \end{aligned}$$

The difference between tax computed in the normal way ( $1,217.76 = 14,760.72 \times 0.0825$ ) and the calculated payment tax (1,327.26) is the **additional tax (109.50) levied on the payment tax.**

The \$1,327.26 includes the additional tax on tax which amounts to \$109.50. Here's why...

We have the tax on the total payments of \$14,760.72...

$$0.08250 \times 14,760.72 = 1,217.76 \dots \text{PLUS, tax on the total payment tax...}$$

$$0.08250 \times 1,327.26 = 109.50 \quad (\text{tax on tax})$$

$$T_B = 1,327.26$$

Apparently, Illinois changed their tax policy around 2015. Title/registration fees, warranties, flat tax, services contracts, negative trade equity capped in a lease, and rebates are all taxable items in most cases. Also, capitalize sales tax SHOULD NOT BE TAXED. Yet, Illinois taxes sales tax as shown above and below.

Sales tax on rebates and cash cap reductions is computed in the same way as above...

$$T_D = (D) \frac{\tau}{1-\tau} . \text{ This gives us...}$$

$$T_D = (1500.00 + 582.15) \frac{0.08250}{1-0.08250}$$

$$= 187.22$$

And, again, we have tax on tax...

$$0.08250 \times 2,082.15 + 0.08250 \times 187.22$$

$$171.78 \quad + \quad 15.44 \quad = \quad 187.22$$

$$\text{Tax} \quad + \quad \text{Tax on Tax}$$

Total Sales Tax Liability:

$$T = T_B + T_D$$

$$= 1,327.26 + 187.22$$

$$= 1,514.48$$

The contractual lease payment is computed as follows:

$$P_L = F(S + A + T - D + R) + \frac{(S + A + T - D - R)}{N}$$

$$P_L = 0.00035(45500 + 1111.94 + 1514.48 - 2082.15 + 30797.10) + \frac{45500 + 1111.94 + 1514.48 - 2082.15 - 30797.10}{39}$$

$$= 417.85$$

So, now we've computed all the calculated values in the dealer's worksheet to the penny...hooray! However, there is a small problem. Because sales tax is capitalized, the fund provider receives the interest levied on the sales tax but, the state does not. Illinois (and NY) require that that they receive the capitalized sales tax interest charge. Therefore, the dealer WS is slightly inaccurate.

Total Payment Sales Tax, including interest levied on the sales tax, is computed as follows:

$$T_B^* = 1327.26 \left( 1 + .00035 \times 39 \times \frac{.0825}{(1-.0825)} \right)$$

$$= 1328.89$$

The 1.63 difference reflects total interest on the capped payment sales tax over the term of the lease.

Cap Reduction Sales Tax, including interest levied on the sales tax, is computed similarly:

$$T_D^* = 187.22 \left( 1 + .00035 \times 39 \times \frac{.0825}{(1 - .0825)} \right)$$

$$= 187.45$$

Total Sales Tax, including interest on sales tax, amounts to:

$$T^* = T_B^* + T_D^*$$

$$= 1328.89 + 187.45$$

$$= 1516.34$$

and not 1514.48 as computed by the dealer.

Now, we'll need to revise the contractual lease payment:

$$P_L = F(S + A + T - D + R) + \frac{(S + A + T - D - R)}{N}$$

$$P_L = 0.00035(45500 + 1111.94 + 1516.34 - 2082.15 + 30797.10) + \frac{45500 + 1111.94 + 1516.34 - 2082.15 - 30797.10}{39}$$

$$= 417.90$$

Note that this payment is slightly higher than the previous calculated payment of 417.85. The reason is that the 417.90 payment accounts for the additional sales tax levied on the interest charge so that it complies with the Illinois sales tax guidelines.

That's a lot of work for little in return! Fortunately, there is a payment formula that enables us to circumnavigate this issue and makes the computation of the payment sales tax much more natural and straightforward. This leads us to Method II:

## METHOD II

Finally, and here is the kicker, we could forego all the aforementioned by simply recognizing that the correct contractual lease payment can be computed using the following formula:

$$P_L^* = \frac{FN(S + A + D\tau - D + R) + (S + A + D\tau - D - R)}{N[1 - \tau(1 + FN)]}$$

$$= \frac{0.00035 \times 39 \times (45500 + 1111.94 + 2082.15 \times .0825 - 2082.15 + 30797.10) + (45500 + 1111.94 + 2082.15 \times .0825 - 2082.15 - 30797.10)}{39 \times [1 - .0825 \times (1 + 0.00035 \times 39)]}$$

$$P_L^* = 417.90$$

Again, this payment is slightly higher than the dealer's payment of 417.85 but does agree with the revised payment computed under Method I. The revised total Sales Tax Liability amounts to:

$$T^* = .0825 \times (417.90 \times 39 + 2082.15)$$

$$= 1516.37$$

This reflects an additional 0.03 from the total sales tax computed under Method I. Apparently, not all dealership desking software compute contractual payments and sales tax the same way. Some may not even comply with portions of the Illinois sales tax guidelines and, of course there are those that do comply as we'll discover in the next example. Fortunately, the difference, in this case, only amounts to pennies but would be significantly more with a higher cost of money and higher payments.



Summarizing this dealer's lease worksheet, we have:

$$\tau = \text{Sales Tax Rate} = 7.00\%$$

$$F = \text{Money Factor} = 0.00145$$

$$N = \text{Term (months)} = 36$$

$$S = \text{Sell Price} = 44,577.00$$

$$A = \text{Capped Fees subject to tax} = 925.00$$

$$C = \text{Capped Fees not subject to tax} = 1,560.21 \text{ (sales tax)}$$

$$T = \text{Total Sales Tax Liability} = 1,560.21$$

$$D = \text{Cap Reduction} = 4500.00 \text{ (rebate)}$$

$$R = \text{Residual Value} = 28,962.60$$

$$P_L = \text{Contractual Monthly Lease Payment} = 481.48$$

$$\text{DAS} = 481.48 + 455.81 = 937.29$$

$$U = \text{DAS fees, other than taxable cap reductions, subject to sales tax} = 455.81$$

As before, we'll calculate the following:

$$P_B = \text{Base Payment (Excludes Sales Tax)} = 378.48$$

$$T_B = \text{Total Payment Sales Tax Liability} = 1,327.26$$

$$T_D = \text{Sales Tax on Cash Cap Reduction and Rebate} = 187.22$$

$$T = \text{Total Sales Tax Liability} = 1,514.48$$

$$P_L = \text{Contractual Monthly Lease Payment}$$

Reminder: The only purpose of the base payment,  $P_B$ , is to compute sales tax.

## METHOD I

$$\text{Base payment (excludes sales tax)} : P_B = F(S + A - D + R) + \frac{(S + A - D - R)}{N}$$

Substituting the assigned values, we get

$$P_B = 435.88$$

Total Base Payments, K:

$$K = P_B N$$

$$= 435.88 \times 36$$

$$= 15,691.68$$

Total base payment sales tax:

$$T_B = (15,691.68) \frac{0.0700}{1 - 0.0700}$$

$$= 1181.09$$

Sales tax on rebates and on taxable DAS is computed as follows...

$$T_D = (D + U) \frac{\tau}{1 - \tau}$$

$$T_D = (4500 + 455.81) \frac{0.0700}{1 - 0.0700}$$

$$= 373.01$$

The Total Sales Tax Liability including interest levied on sales tax amounts to:

$$\begin{aligned}
 T &= (T_B + T_D) \left( 1 + FN \frac{\tau}{1 - \tau} \right) \\
 &= (1181.09 + 373.01) \left( 1 + .00145 \times 36 \times \frac{.07}{.93} \right) \\
 &= 1560.21
 \end{aligned}$$

Finally, compute the contractual lease payment:

$$\begin{aligned}
 P_L &= F(S + A + T - D + R) + \frac{(S + A + T - D - R)}{N} \\
 &= .00145 \times (44577 + 925 + 1560.21 - 4500 + 28962.60) + \frac{(44577 + 925 + 1560.21 - 4500 - 28962.60)}{36} \\
 &= 481.48
 \end{aligned}$$

The dealer appears to have complied with the Illinois sales tax guidelines. No need to resort to Method II but let's do it anyway just to see how close it gets us to the dealer's calculations.

## METHOD II

$$\begin{aligned}
 P_L^* &= \frac{FN(S + A + (D + U)\tau - D + R) + (S + A + (D + U)\tau - D - R)}{N[1 - \tau(1 + FN)]} \\
 &= \frac{.00145 \times 36 \times (44577 + 925 + 4955.81 \times .07 - 4500 + 28962.60) + (44577 + 925 + 4955.81 \times .07 - 4500 - 28962.60)}{36 \times [1 - .07 \times (1 + .00145 \times 36)]} \\
 &= 481.48
 \end{aligned}$$

Note that, unlike Example I, this contractual payment calculation agrees exactly with the dealer's contractual payment calculation. The reason is that the dealer's calculations comply with the Illinois sales tax guideline. And so, the total Sales Tax Liability that could be remitted to the Illinois DMV amounts to:

$$\begin{aligned}
 T^* &= .07 \times (481.48 \times 36 + 4955.81) \\
 &= 1560.24
 \end{aligned}$$

Observe that the dealer's calculated total sales tax liability of 1560.21 only differs by 0.03 from the 1560.24 just calculated and, even though 0.03 reflects significant rounding error, the percent of accuracy is 99.998% which is more than acceptable as we used two different computational methods where slight differences are to be expected. As such, the calculated sales tax under either method can be submitted to the Illinois DMV. I doubt they're going to quibble over a few pennies especially with an accuracy of nearly 100%. If you have access to multiple desking software products and are observing differences of a few dollars or more, then something is wrong. Desking software packages should be spot on or within a few pennies at most.

### EXAMPLE III

The following dealer worksheet reflects sales tax that is not capitalized. Okay, so I lied when I said we'll only examine those leases that capitalize sales tax. But I decided to cover all the bases in the interests of thoroughness.

FI9D0K	Lease Information Screen		ZBO-FI
Deal Number:	61434	17) Initial Cap Cost:	\$ 95,509.00
1) Contract Date:	02/28/24	18) Total Insurance:	
2) First Payment Date:	03/30/24	19) Total Add Cap Costs:	\$ 1,283.03
3) Lease Institution:	BMWLSE	20) Cash Cap Reduction:	\$ 1,449.52
		21) Total Trade Allow:	
4) M.S.R.P.:	\$ 104,955.00	22) Rebate Amount:	\$ 9,900.00
5) Add To MSRP w/Markup:		Adjusted Cap Cost:	\$ 88,638.44
6) Mileage Penalty:		Base Monthly Rental:	\$ 899.48
7) L.E.V.%:	54.00%	23) Security Deposit:	\$ 6,300.00
8) Lease-end Value:	\$ 56,675.70	24) Total Annual Fees:	\$ 316.00
9) Term:	36	25) Total Initial Fees:	\$ 35.00
10) Money Fact Sell Rate:	0.00008	26) One Pay(Y/N)/Amt: N	
11) Customer Cash Down:	\$ 9,000.00	27) CapTax1 Rt/Amt:	
12) Credit Life/A&H Code:	NO	28) CapTax2 Rt/Amt:	
13) Total Sales Tax:	\$ 3,195.93	29) Amount Due At Start:	\$ 9,000.00
14) Tot CL On Mthly Pay:		30) Total Working Cash:	\$ 9,000.00
15) AD Valorem Tax .00000%		31) Total Monthly Payment:	\$ 899.48
16) Purch Option Fee:	\$ 300.00	32) Disposition Fee:	\$ 495.00
Command:			
F1=Help    F2=Home    F3=Save    F4=Cancel			

Summarizing the dealer's lease worksheet, we have:

$\tau$  = Sales Tax Rate = 7.250%

$F$  = Money Factor = 0.00008

$N$  = Term (months) = 36

$S$  = Sell Price = 95,509.00.

$A$  = Capped Fees subject to tax = 925.00 (acquisition fee)

$C$  = Capped Fees not subject to tax = 358.03 (license/title/registration)

$D$  = Cash Cap Reduction plus Rebate Cap Reduction = 1,449.52 (cash) + 6704.07 (rebate) = 8,153.59

$B$  = Rebate = 9,900.00 (subject to sales tax)

$T$  = Total Sales Tax Liability = 3,195.93 (paid with rebate bal. 9,900.00 – 6704.07)

$R$  = Residual Value = 54% x 104,955.00 = 56,675.70

$Q$  = Adjusted Capitalized Cost = 95,509.00 - 8,153.59 + 1,283.03 = 88,638.44

$P_L$  = Contractual Monthly Lease Payment = 899.48

$MSD$  = 7 @900.00 each = 6,300.00

$DAS$  = 899.48 + 1,449.52 + 350 (Doc) + 6,300.00 = 9000,00

$U$  = DAS fees, other than taxable cap reductions, subject to sales tax = 350.00



We'll calculate the following:

$P_L$  = Contractual Lease Payment

$T$  = Total Sales Tax Liability

$$P_L = .00008 \times (88638.44 + 56675.70) + \frac{(88638.44 - 56675.70)}{36}$$

$$= 899.48$$

$$T = 7.25\% \times (899.48 \times 36 + 9900.00 + 1449.52 + 350.00)$$

$$= 3195.93$$

Note 1: The base payment,  $P_B$ , is identical to  $P_L$ .

Note 2: Nowhere is the \$35 initial fee considered.

In summary, there are two methods to compute total sales tax liability when capitalizing sales tax in Illinois.

#### Method I:

$$T = \frac{\tau}{1-\tau} (P_B N + D + U) \left( 1 + FN \frac{\tau}{1-\tau} \right)$$

$T$  = Total Sales Tax Liability

$P_B$  = Base Payment

$F$  = Money Factor

$N$  = Term

$D$  = Capitalized Cost Reduction

$U$  = Taxable (Upfront) DAS Fees (excludes: sales tax, 1st payment, taxable cap reductions, some fees)

$\tau$  = Sales Tax Rate

$$P_L = F(S + A + T - D + R) + \frac{(S + A + T - D - R)}{N} \quad (\text{contractual payment})$$

$S$  = Sell Price

$A$  = Capitalized Fees subject to sales tax

$R$  = Residual Value

#### Method II:

$$T^* = \tau (P_L^* N + D + U)$$

$T^*$  = Total Sales Tax Liability

$N$  = Term

$D$  = Capitalized Cost Reduction

$U$  = Taxable (Upfront) DAS Fees (excludes: sales tax, 1st payment, taxable cap reductions, some fees)

$\tau$  = Sales Tax Rate

$$P_L^* = \frac{FN(S + A + (D + U)\tau - D + R) + (S + A + (D + U)\tau - D - R)}{N[1 - \tau(1 + FN)]} \quad (\text{contractual payment})$$

A word of caution. Not all leases are created equal. You should organize the dealer's data similar to what was done above or, better yet, organize your own data based upon your own research. Recognize that some of your data or your dealer's data may be different or in addition to what has been described above and so, you'll need to determine whether it should be incorporated into your calculations. You'll need to think as there is no round peg, round hole approach to leasing. Perform all pertinent calculations and then, create a one-page professional-looking lease proposal and email it to a decision-maker at the dealership. DO NOT negotiate inside of a dealership. Let your lease proposal and all your hard work speak on your behalf and do some tweaking if necessary until you have achieved a fabulous deal.

## APPENDIX

The following only applies to the calculation of sales tax using Method I. It is assumed that the only sales tax is the sales tax levied on the sum of the base payments and capitalized in the lease. Method I provides a methodology of computing sales tax,  $T_B$ , only this time  $T_B = T$  so that

$$P_L N \tau = T \quad (1)$$

In order for this equation to hold true, the sales tax must be computed as follows

$$P_B N \frac{\tau}{1-\tau} \left( 1 + FN \frac{\tau}{1-\tau} \right) = T \quad (2)$$

So that

$$P_L N \tau = P_B N \frac{\tau}{1-\tau} \left( 1 + FN \frac{\tau}{1-\tau} \right) \quad \text{Q.E.D.} \quad (3)$$

There is a statement on P. 24 of the Illinois Sales Tax Guideline (ST-9-LSE, (2/23)) that reads...

*This formula only works if you actually use this as the "Total of Payments" amount in the lease contract*  
As such, our goal is to prove equation (3).

Let

$$P_B = F(C + R) + \frac{C - R}{N}$$

and

$$P_L = F(C + T + R) + \frac{C + T - R}{N}$$

Equating the left-hand sides of equations (1) & (2), gives us equation (3)

$$P_L N \tau = P_B N \frac{\tau}{1-\tau} \left( 1 + FN \frac{\tau}{1-\tau} \right)$$

Substituting, we get

$$\left[ F(C + T + R) + \frac{C + T - R}{N} \right] N \tau = T$$

Solve for  $T$

$$\begin{aligned}
T &= \frac{[FN(C + R) + C - R]\tau}{1 - \tau(1 + FN)} \\
&= \frac{P_B N \tau}{1 - \tau(1 + FN)} \\
&= \frac{P_B N \tau}{\frac{[1 - \tau(1 + FN)](1 - \tau)}{1 - \tau}} \\
T &= P_B N \frac{\tau}{1 - \tau} \left[ \frac{1 - \tau}{1 - \tau(1 + FN)} \right] \quad (4)
\end{aligned}$$

Referencing equations (2) & (4), we need to show that

$$\begin{aligned}
1 + FN \frac{\tau}{1 - \tau} &= \frac{1 - \tau}{1 - \tau(1 + FN)} \\
&= \frac{1}{\frac{1}{1 - \tau} - \frac{\tau}{1 - \tau}(1 + FN)} \\
&= \frac{1}{\frac{1}{1 - \tau} - \frac{\tau}{1 - \tau} - FN \frac{\tau}{1 - \tau}} \\
&= \frac{1}{1 - FN \frac{\tau}{1 - \tau}} \\
&= \frac{1 + FN \frac{\tau}{1 - \tau}}{\left(1 + FN \frac{\tau}{1 - \tau}\right) \left(1 - FN \frac{\tau}{1 - \tau}\right)} \\
&= \frac{1 + FN \frac{\tau}{1 - \tau}}{1 - \left(FN \frac{\tau}{1 - \tau}\right)^2}
\end{aligned}$$

Note

$$\left(FN \frac{\tau}{1 - \tau}\right)^2 \approx 0, \quad 0 \leq F < 1, \quad N > 0, \quad 0 \leq \tau < 1$$

And, as a matter of practicality, it is equal to zero to the order of  $1 \times 10^{-5}$  for most applications. Therefore

$$1 + FN \frac{\tau}{1 - \tau} = \frac{1 - \tau}{1 - \tau(1 + FN)}$$

So that

$$\begin{aligned} T &= P_B N \frac{\tau}{1 - \tau} \left[ \frac{1 - \tau}{1 - \tau(1 + FN)} \right] \\ &= P_B N \frac{\tau}{1 - \tau} \left[ 1 + FN \frac{\tau}{1 - \tau} \right] \end{aligned}$$

and

$$P_L N \tau = P_B N \frac{\tau}{1 - \tau} \left[ 1 + FN \frac{\tau}{1 - \tau} \right]$$